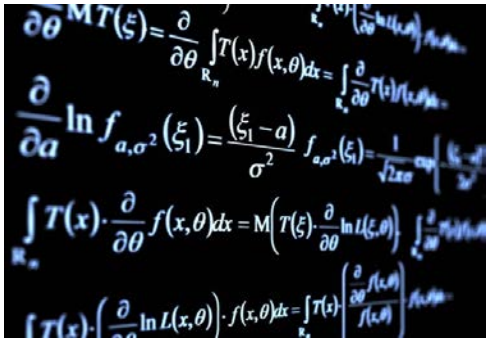

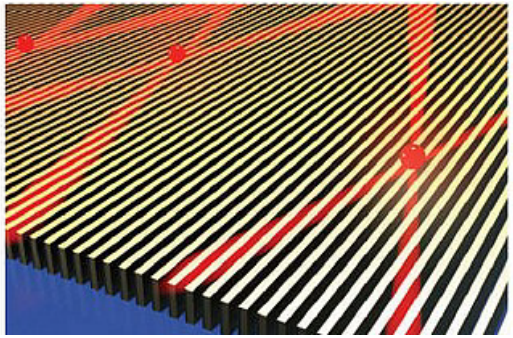

Spatial Symmetries in Electromagnetics and Properties of Higher-Order Symmetries

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COST ACTION CA18223

SYMAT

FUTURE COMMUNICATIONS WITH HIGHER-SYMMETRIC ENGINEERED ARTIFICIAL MATERIALS

GeePS

Génie électrique et électronique de Paris

SORBONNE UNIVERSITÉ

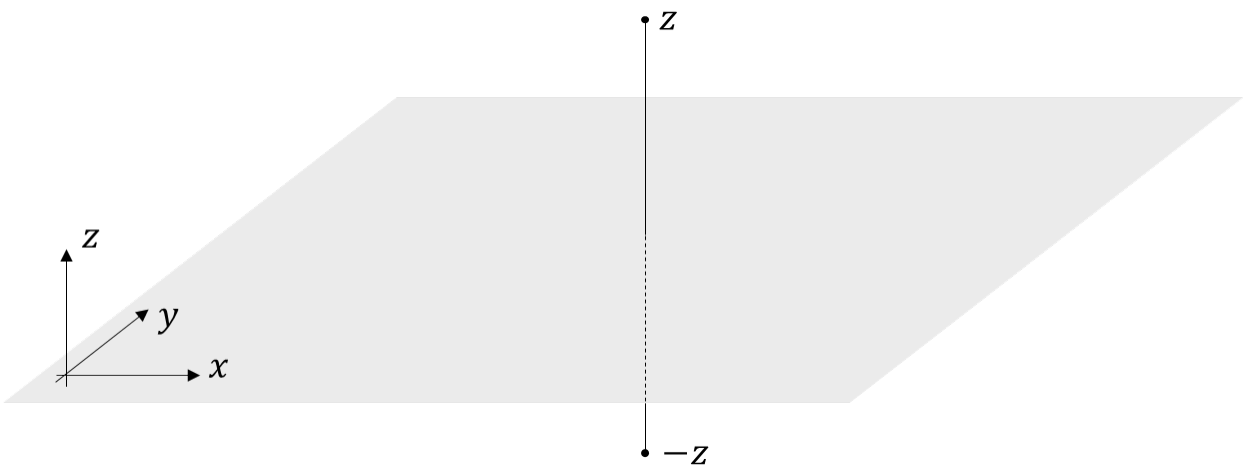
1

Simple Spatial Symmetries and Maxwell Equations

Symmetry with Respect to a Plane

Very common geometrical symmetries in electromagnetics are defined with respect to a plane, here (x, y)

Let a structure be invariant under the mirroring around the $z = 0$ plane: $M: \begin{cases} x \to x \\ y \to y \\ z \to -z \end{cases}$



Symmetry with Respect to a Plane

The modal fields of the structures are solutions of the eigenproblem

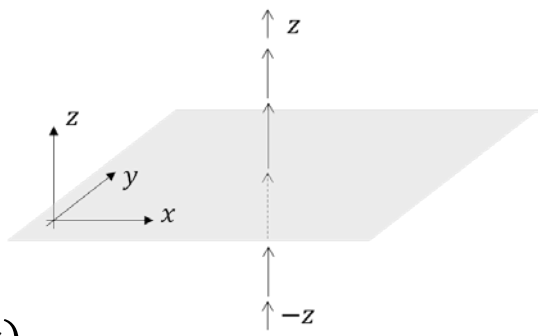
$$ME_i = \lambda E_i$$

Since M^2 is the identity operator, $\lambda = \pm 1$

$$ME_z = E_z$$

$$\Updownarrow$$

$$E_z(-z) = E_z(z)$$

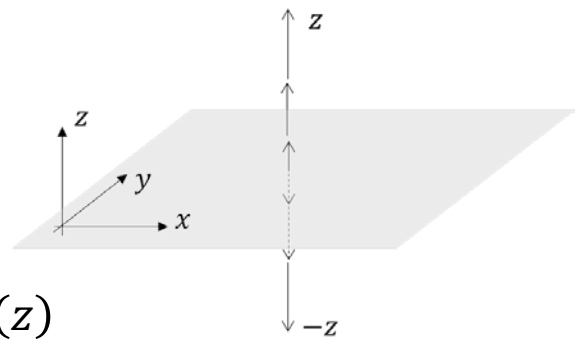


Even E_z field
(the $z = 0$ plane is an equivalent PEC)

$$ME_z = -E_z$$

$$\Updownarrow$$

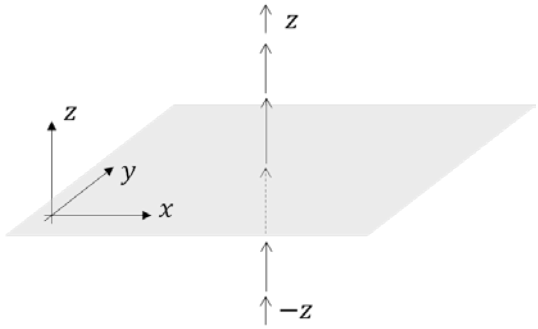
$$E_z(-z) = -E_z(z)$$



Odd E_z field
(the $z = 0$ plane is an equivalent PMC)

Symmetry with Respect to a Plane: PEC

$ME_z = E_z$ Even E_z field



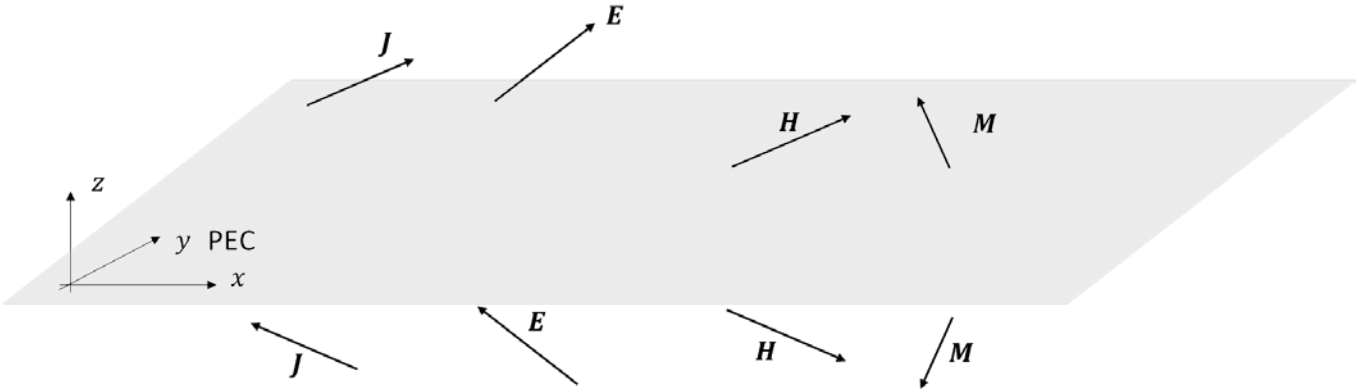
If E_z is even, Maxwell equations are satisfied by

Even H_x, H_y $\quad MH_x = H_x, MH_y = H_y$

Odd E_x, E_y, H_z $\quad ME_x = -E_x, ME_y = -E_y, MH_z = -H_z$

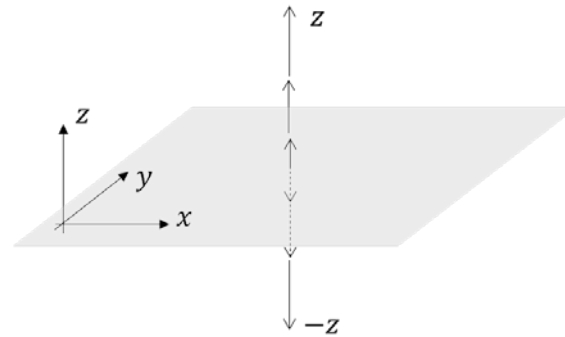
so $E_x(z=0) = 0, E_y(z=0) = 0$ ⟹ $z = 0$ is a PEC plane

If electric (magnetic) sources have the same parity as E (H) fields, only this parity is excited: the $z = 0$ plane is equivalent to a PEC



Symmetry with Respect to a Plane: PMC

$ME_z = -E_z$ Odd E_z field



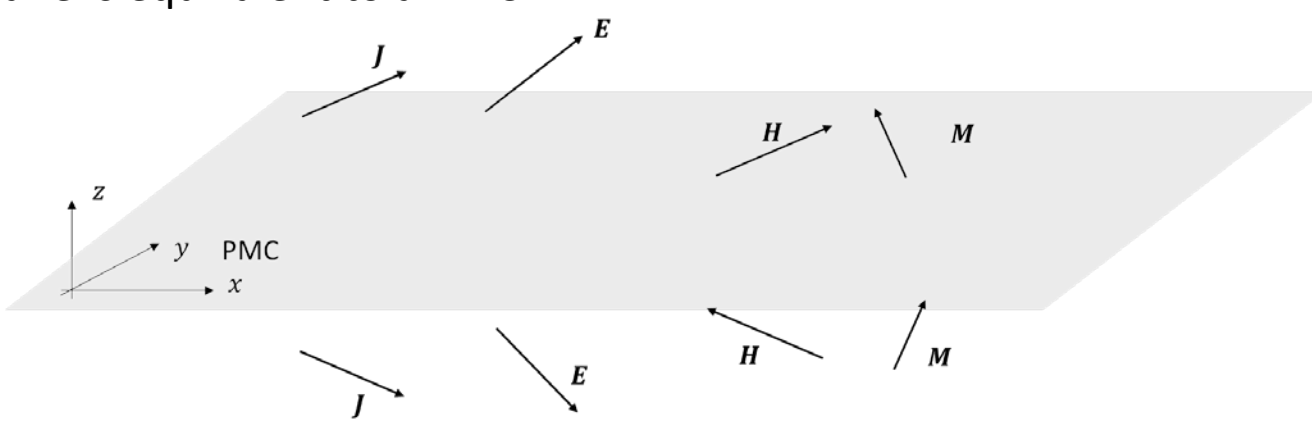
If E_z is odd, Maxwell equations are satisfied by

Odd H_x, H_y $MH_x = -H_x, MH_y = -H_y$

Even E_x, E_y, H_z $ME_x = E_x, ME_y = E_y, MH_z = H_z$

so $H_x(z=0) = 0, H_y(z=0) = 0$ ⟹ $z = 0$ is a PMC plane

If electric (magnetic) sources have the same parity as E (H) fields, only this parity is excited: the $z = 0$ plane is equivalent to a PMC

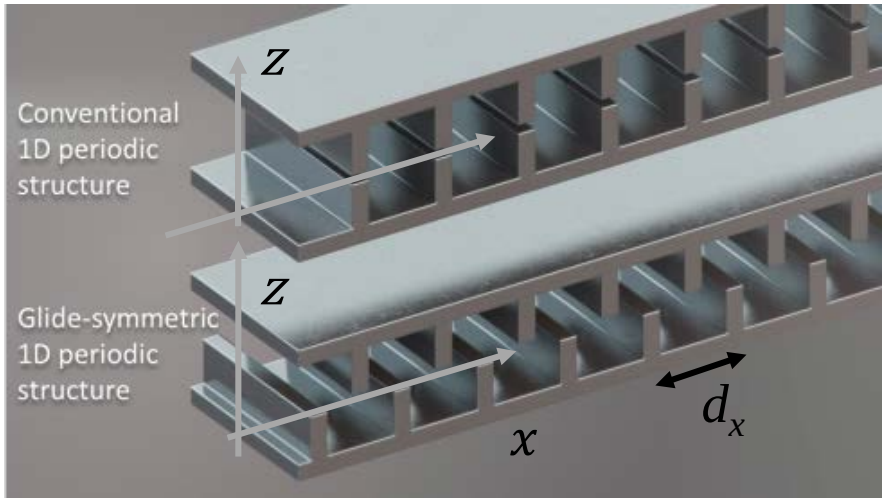


2

Higher Order Spatial Symmetries in Periodic Structures

Glide-Symmetry Along One Direction

A glide symmetric structure is the invariant under the composition of
- a **mirroring**
- a **shift of half a period** in a periodic structure



1-D glide operator

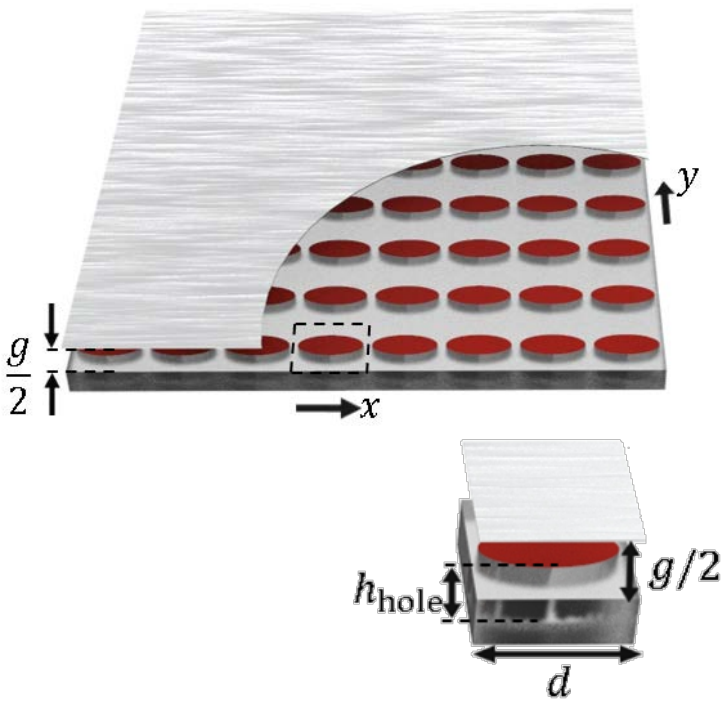
$$G: \begin{cases} x \to x + d_x/2 \\ y \to y \\ z \to -z \end{cases}$$

The symmetry plane $z = 0$ is called the glide plane.

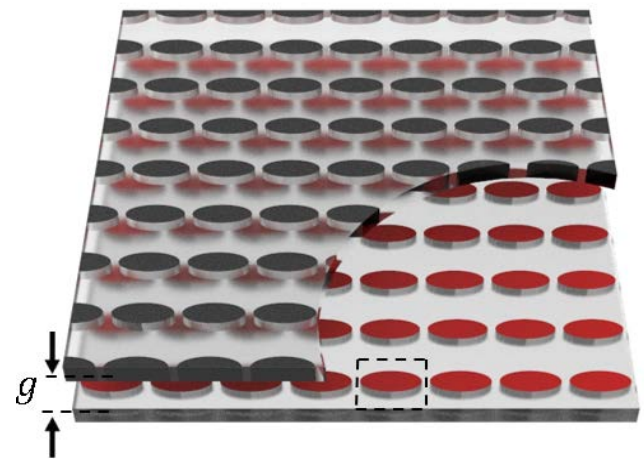
Since two glide transformations are a translation of d_x, a glide structure is also periodic.

Glide-Symmetry Along Two Directions

Conventional (Mirror-Symmetric)

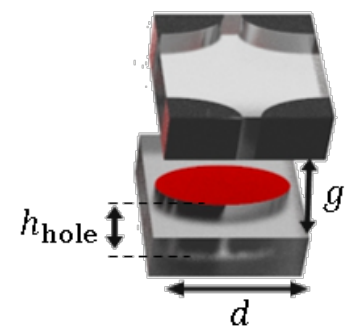


Glide-Symmetric



2-D glide operator

$$G: \begin{cases} x \to x + d_x/2 \\ y \to y + d_y/2 \\ z \to -z \end{cases}$$



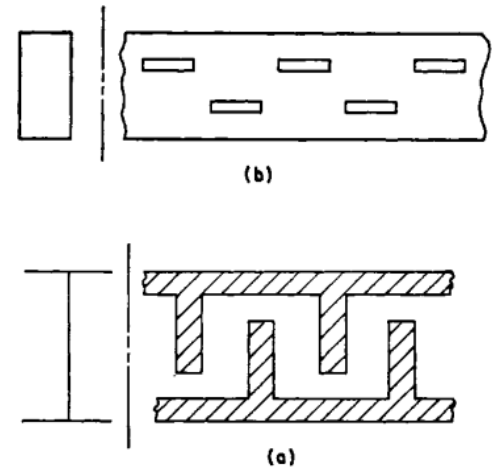
A. Alex-Amor, F. Ghasemifard, G. Valerio, M. Ebrahimpouri, P. Padilla, F. Gonzalez, J. Manuel, and O. Quevedo-Teruel, "Glide-symmetric metallic structures with elliptical holes for lens compression," *IEEE Trans. Microw. Theory. Tech*. vol. 68, no. 10, Oct. 2020.

1D Glide-Symmetric Structures: Eigenproblem Formulation

PROCEEDINGS OF THE IEEE, VOL. 61, NO. 2, FEBRUARY 1973 183

Propagation in Periodically Loaded Waveguides with Higher Symmetries

ALEXANDER HESSEL, MING HUI CHEN, ROBERT C. M. LI, AND ARTHUR. A. OLINER

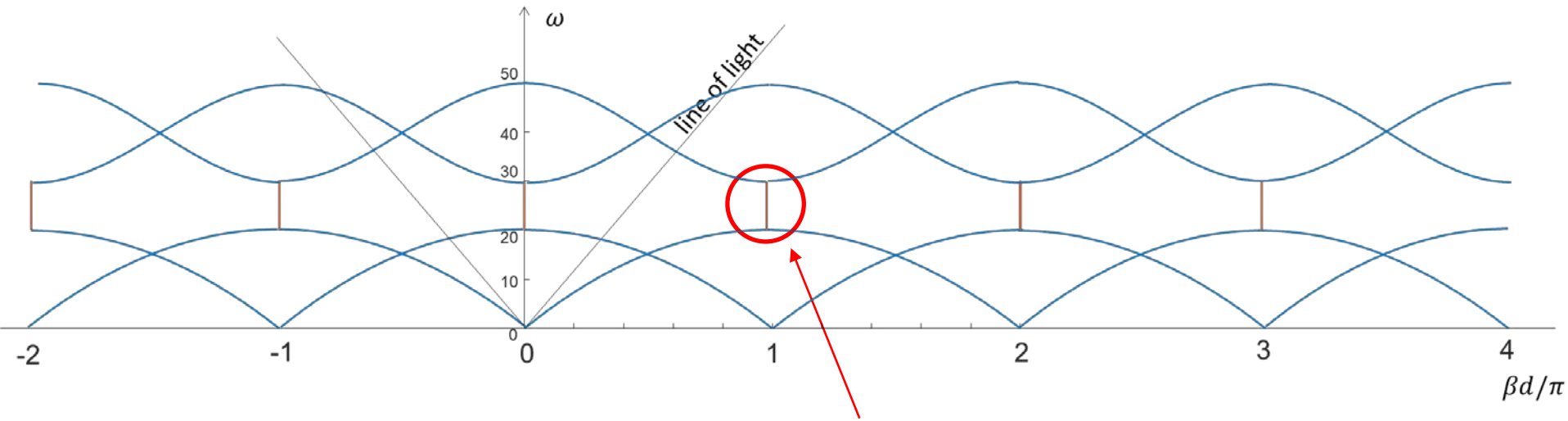


The glide operator G, not the translation operator *T*, is the minimal symmetry of the structure and *determines its dispersion properties*

$$GE_i = \lambda E_i \quad \Rightarrow \quad TE_i = G^2 E_i = \lambda^2 E_i \quad \Rightarrow \quad \lambda = \pm e^{-jk_x \frac{d_x}{2}}$$

Brillouin Diagram of Non-Glide Symmetric Structures

$$TE_i = e^{-jk_x d_x} E_i \longrightarrow k_{x,n} = k_x + \frac{2\pi n}{d_x}$$

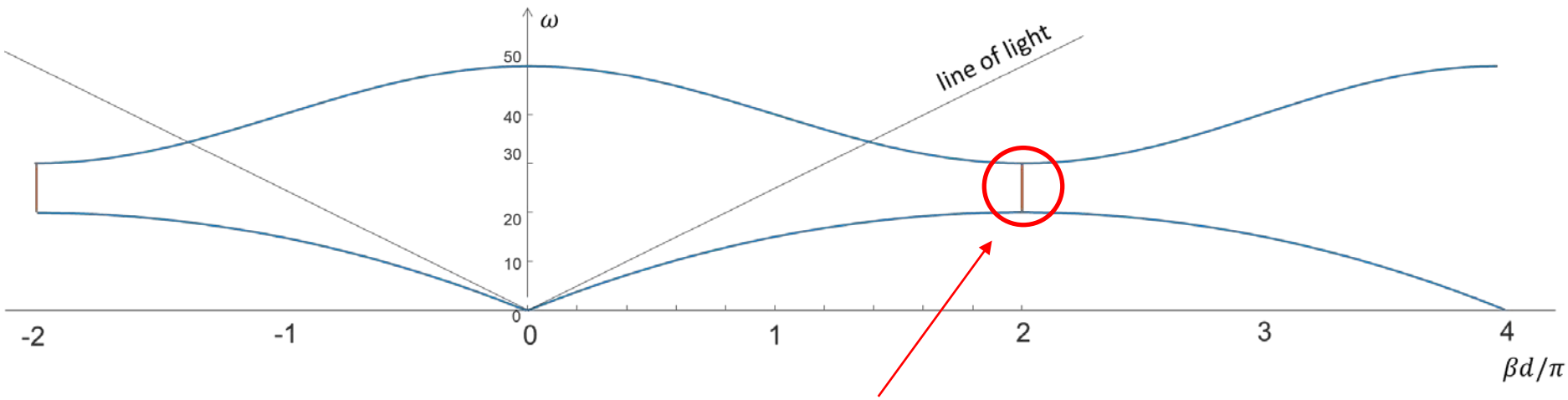


Different harmonics couple when meeting at the same frequency

Stop bands at $\beta d = n\pi$, n odd and even

Brillouin Diagram of Glide Symmetric Structures

$$GE_i = +e^{-jk_x\frac{d_x}{2}}E_i \quad \Longrightarrow \quad k_{x,n}^{(1)} = k_x + \frac{4\pi n}{d_x} \qquad (1)$$

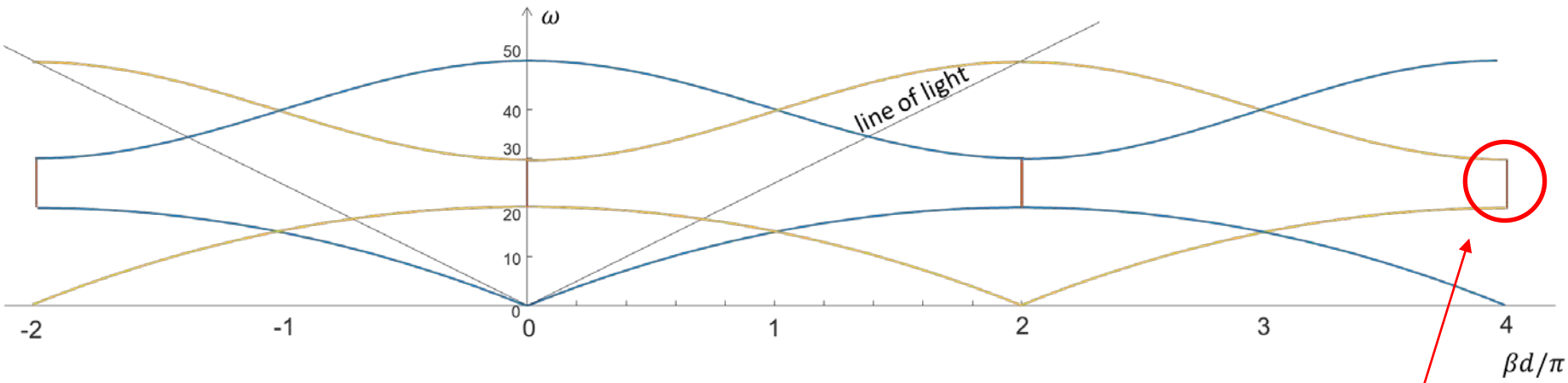


Different harmonics (1) couple only at $\beta d = n\pi$, $n = \pm 2, \pm 6, \ldots$

Brillouin Diagram of Glide Symmetric Structures

$$GE_i = +e^{-jk_x\frac{d_x}{2}}E_i \longrightarrow k_{x,n}^{(1)} = k_x + \frac{4\pi n}{d_x} \qquad (1)$$

$$GE_i = -e^{-jk_x\frac{d_x}{2}}E_i \longrightarrow k_{x,n}^{(2)} = k_{x,n}^{(1)} + \frac{2\pi n}{d_x} \qquad (2)$$



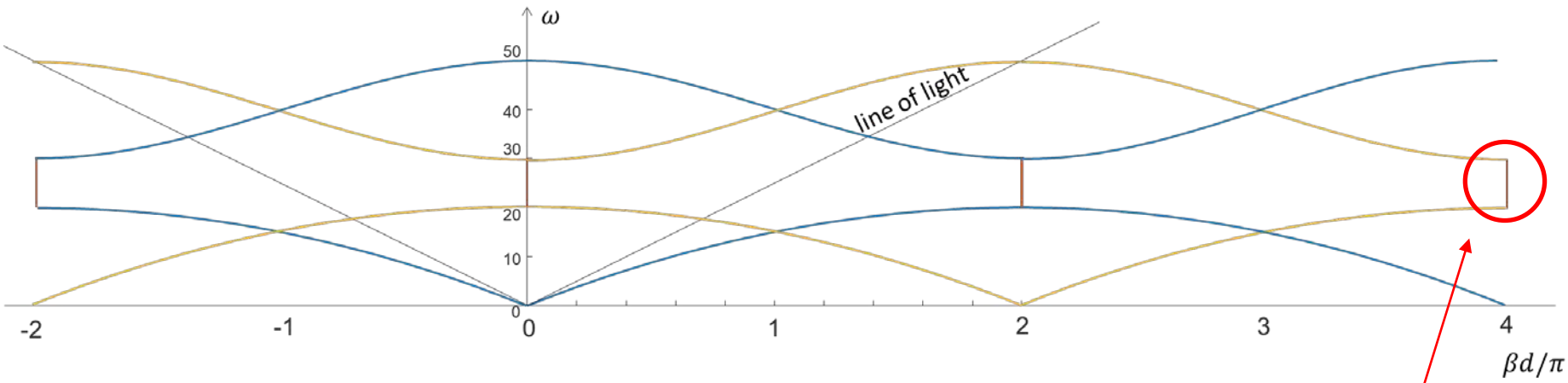
Different harmonics (1) couple only at $\beta d = n\pi$, $n = \pm 2, \pm 6, \pm 10, \ldots$

Different harmonics (2) couple only at $\beta d = n\pi$, $n = \pm 4, \pm 8, \pm 12, \ldots$

Brillouin Diagram of Glide Symmetric Structures

$$GE_i = +e^{-jk_x\frac{d_x}{2}}E_i \longrightarrow k_{x,n}^{(1)} = k_x + \frac{4\pi n}{d_x} \qquad (1)$$

$$GE_i = -e^{-jk_x\frac{d_x}{2}}E_i \longrightarrow k_{x,n}^{(2)} = k_{x,n}^{(1)} + \frac{2\pi n}{d_x} \qquad (2)$$



Different harmonics (1) couple only at $\beta d = n\pi$, $n = \pm 2, \pm 6, \pm 10, \ldots$

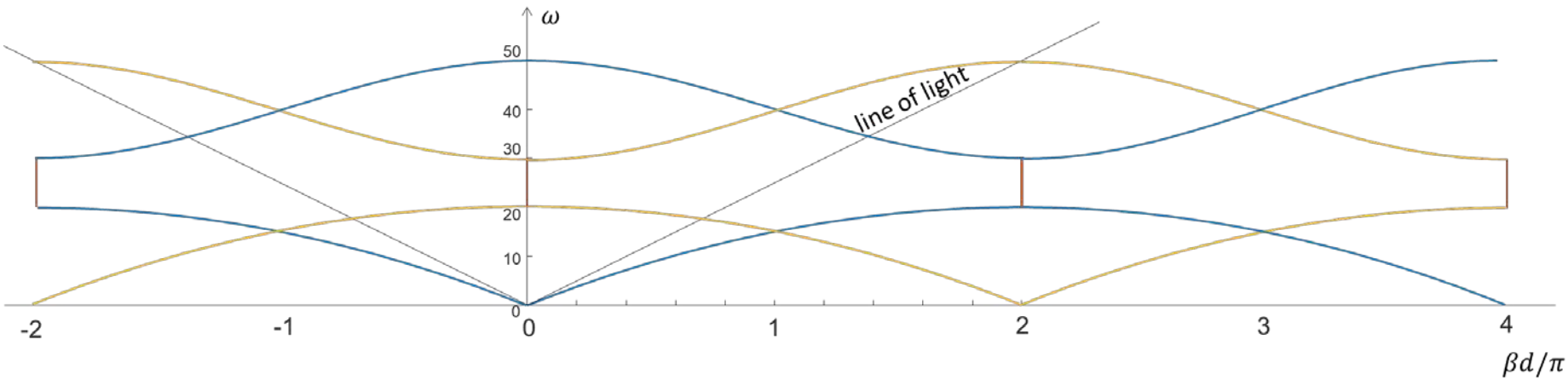
Different harmonics (2) couple only at $\beta d = n\pi$, $n = \pm 4, \pm 8, \pm 12, \ldots$

Harmonics (1) and (2) do not couple since they have different z-symmetry! (to be seen later...)

Brillouin Diagram of Glide Symmetric Structures

$$GE_i = +e^{-jk_x\frac{d_x}{2}}E_i \longrightarrow k_{x,n}^{(1)} = k_x + \frac{4\pi n}{d_x} \quad (1)$$

$$GE_i = -e^{-jk_x\frac{d_x}{2}}E_i \longrightarrow k_{x,n}^{(2)} = k_{x,n}^{(1)} + \frac{2\pi n}{d_x} \quad (2)$$



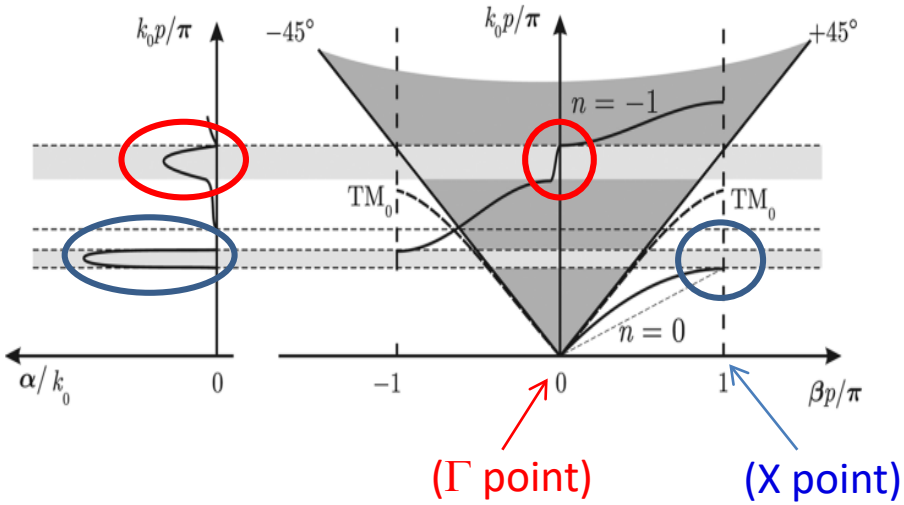
Glide-symmetric structures only have stop bands at $\beta d = n\pi$, n even

No stop band is present at $\beta d = \pi$ (closed stop-band, X point)

Closed and Open Stopbands (Γ and X Points)

The **absence of a stop-band** at $\beta d = \pi$ (X point) is typical of glide-symmetric structures.

Analogous to the open-stop band suppression at $\beta = 0$ for broadside-radiating leaky waves.



2826 IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, VOL. 56, NO. 12, DECEMBER 2008

Full-Wave Modal Dispersion Analysis and Broadside Optimization for a Class of Microstrip CRLH Leaky-Wave Antennas

Simone Paulotto, *Member, IEEE*, Paolo Baccarelli, *Member, IEEE*, Fabrizio Frezza, *Senior Member, IEEE*, and David R. Jackson, *Fellow, IEEE*

Symmetry Properties of Harmonics 1/2

$$GE_i = +e^{-jk_x\frac{d_x}{2}}E_i \longrightarrow k_{x,n}^{(1)} = k_x + \frac{4\pi n}{d_x} \quad (1)$$

$$GE_i = -e^{-jk_x\frac{d_x}{2}}E_i \longrightarrow k_{x,n}^{(2)} = k_{x,n}^{(1)} + \frac{2\pi n}{d_x} \quad (2)$$

We can associate

- solutions (1) to harmonics having even integer n. Note that $e^{-jk_{x,n}\frac{d_x}{2}} = +e^{-jk_x\frac{d_x}{2}}$
- solutions (2) to harmonics having odd integer n. Note that $e^{-jk_{x,n}\frac{d_x}{2}} = -e^{-jk_x\frac{d_x}{2}}$

So we can state $e^{-jk_{x,n}\frac{d_x}{2}} = (-1)^n e^{-jk_x\frac{d_x}{2}}$

Symmetry Properties of Harmonics 2/2

Remember that $e^{-jk_{x,n}\frac{d_x}{2}} = (-1)^n e^{-jk_x\frac{d_x}{2}}$

Let us apply the glide operator to a Bloch mode:

$$GE_i = G\sum_{n=-\infty}^{+\infty} c_n(z)e^{-jk_{x,n}x} = \sum_{n=-\infty}^{+\infty} c_n(-z)e^{-jk_{x,n}\left(x+\frac{d_x}{2}\right)} = \sum_{n=-\infty}^{+\infty} c_n(-z)e^{-jk_{x,n}x}e^{-jk_{x,n}\frac{d_x}{2}}$$

$$= \sum_{n=-\infty}^{+\infty} c_n(-z)e^{-jk_x x}e^{-jk_{x,n}\frac{d_x}{2}} = \sum_{n=-\infty}^{+\infty} c_n(-z)e^{-jk_{x,n}x}(-1)^n e^{-jk_x\frac{d_x}{2}}$$

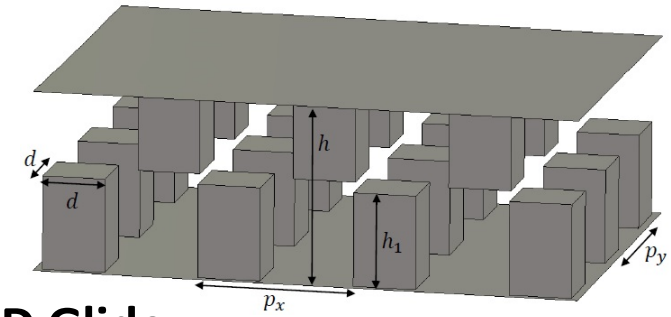
$$= e^{-jk_x\frac{d_x}{2}}\sum_{n=-\infty}^{+\infty}(-1)^n c_n(-z)e^{-jk_{x,n}x} = e^{-jk_x\frac{d_x}{2}}E_i$$

only if $c_n(z) = (-1)^n c_n(-z)$

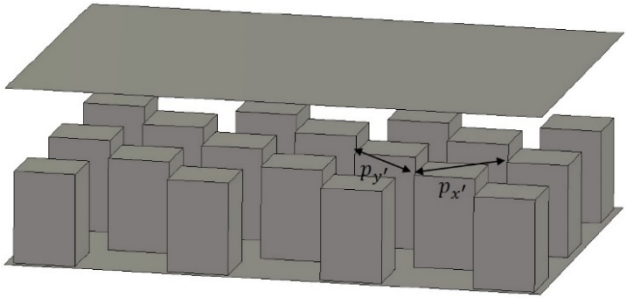
Result 1: Harmonics with different n-parity have different z-parity and do not couple.

Result 2: The glide plane is equivalent to a PEC or PMC according to the n-parity of harmonics.

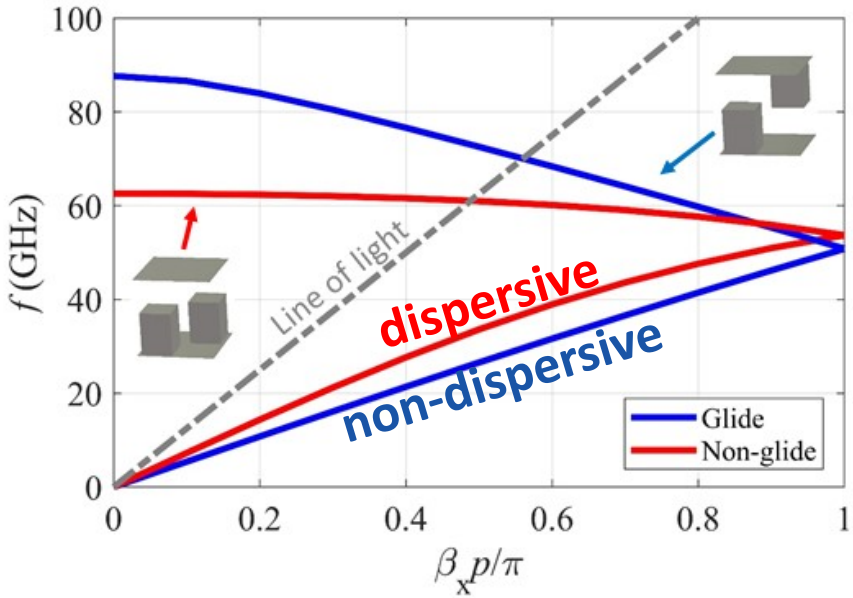
Glide Symmetry vs. Non-Glide Symmetry: Dispersion



2-D Glide

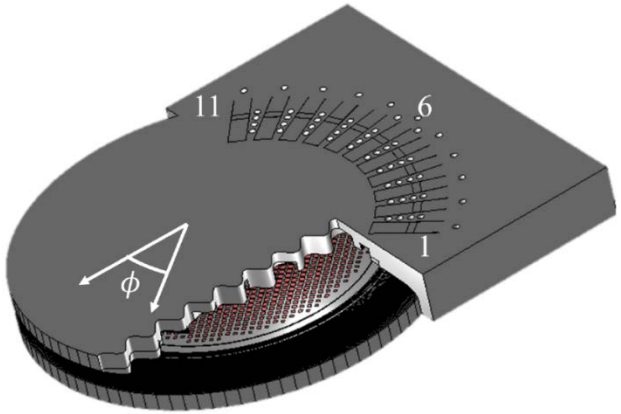


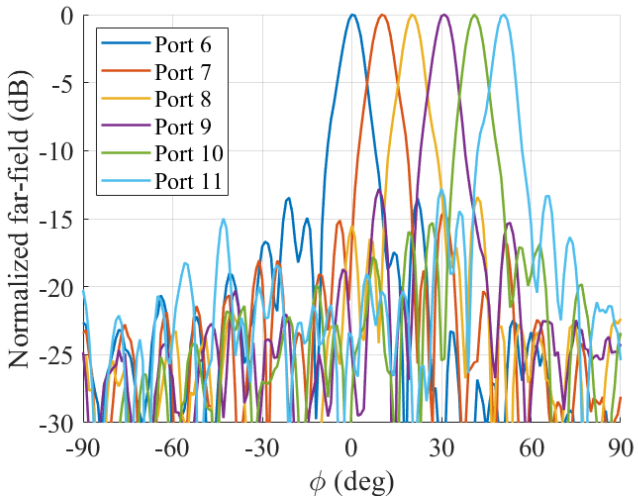
2-D non-glide



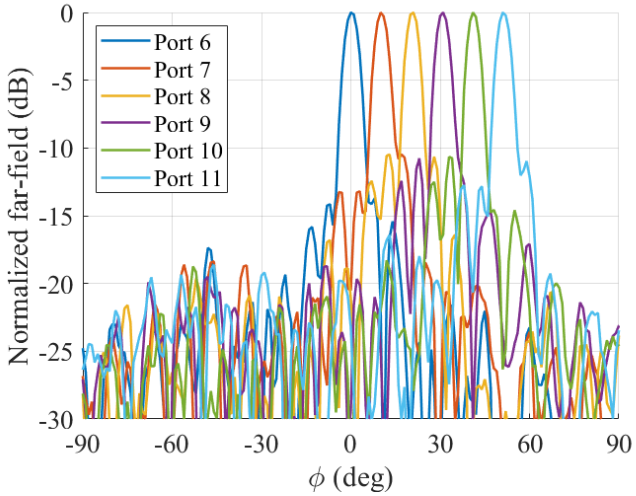
Glide structures are *less dispersive in frequency* than non-glide ones!

Application: Luneburg Lens with Glide-Symmetric Holes





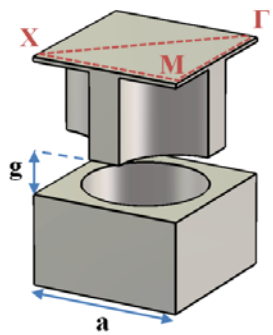
27 GHz



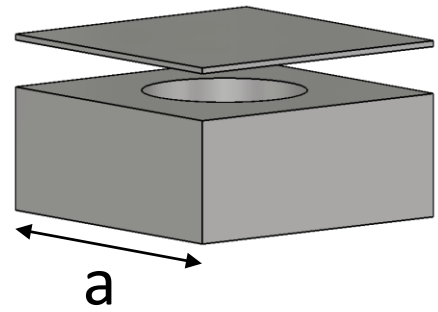
30 GHz

R. Hamarneh, O. Zetterstrom, and O. Quevedo-Teruel, "Glide-Symmetric Luneburg Lens Using Substrate-Integrated-Holes for 5G Communications at Ka-Band", *14th European Conference on Antennas and Propagation* (*EUCAP 2020*), Copenhague, Denmark, 2020.

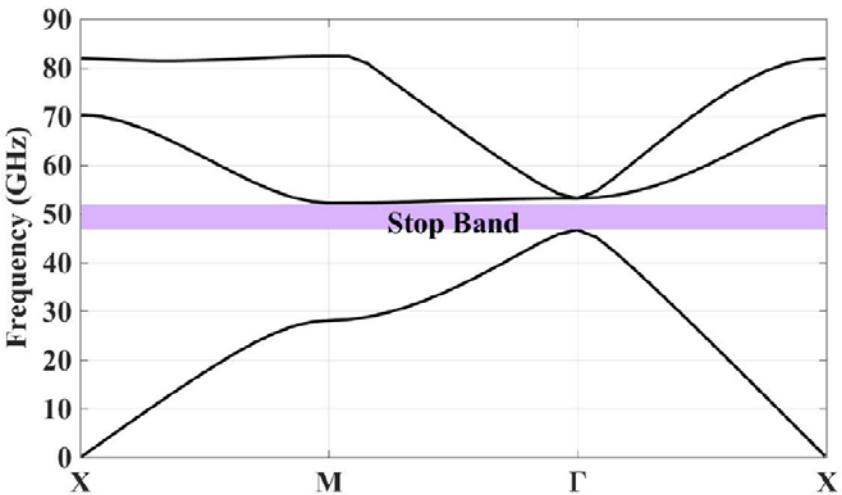
Glide Symmetry vs. Non-Glide Symmetry: Stopband

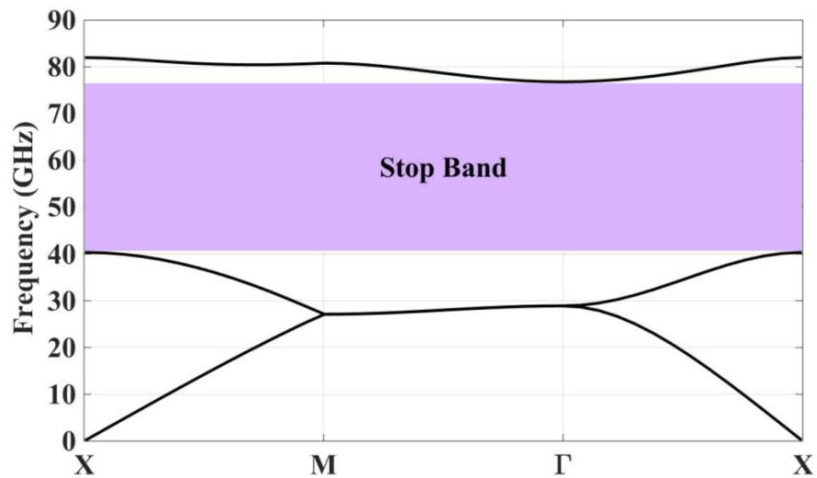


2-D Glide



2-D non-glide

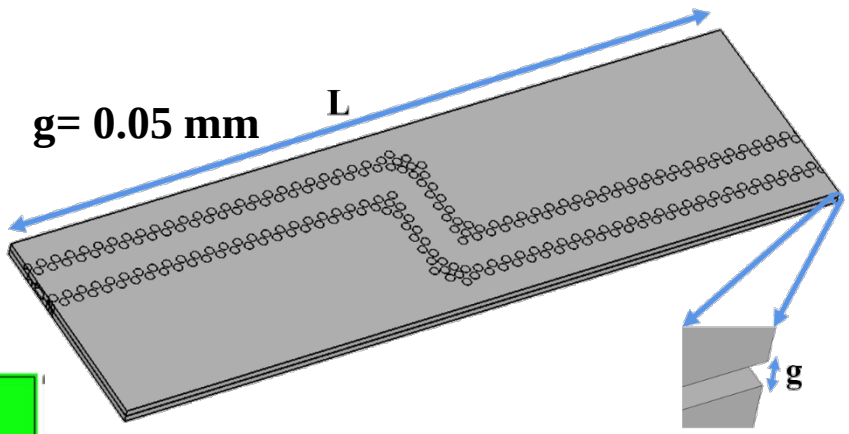


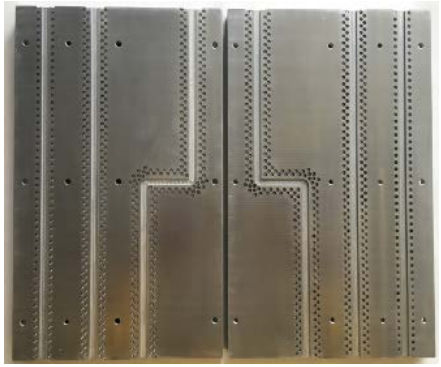


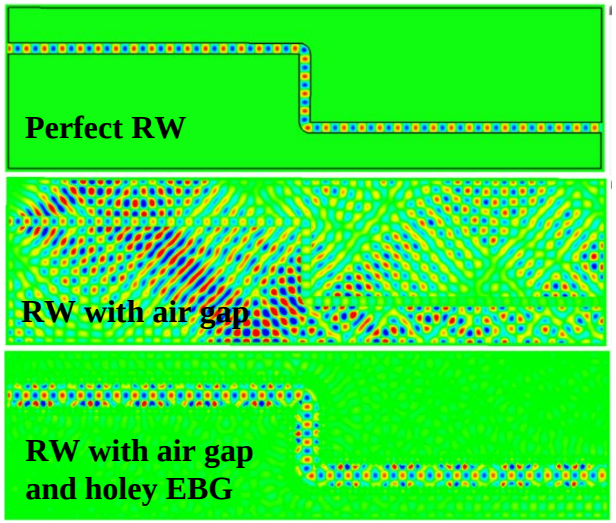
Glide structures have a wider stopband than non-glide ones!

Application: EBG for Integrated Waveguides

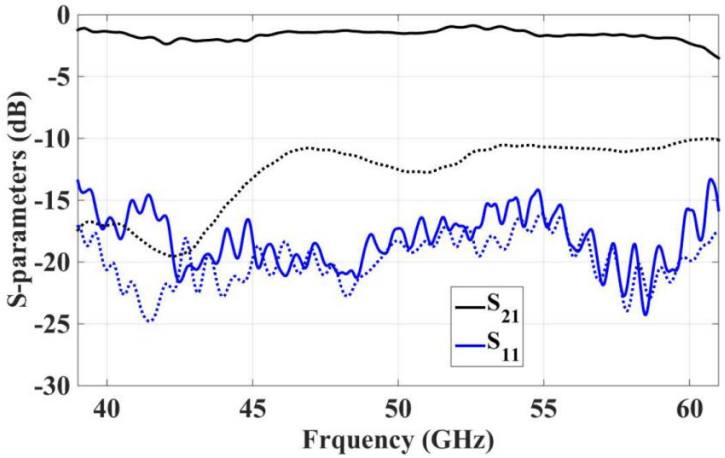
- This technology can be employed to design a number of electromagnetic components:
  - Flanges.
  - Waveguides.





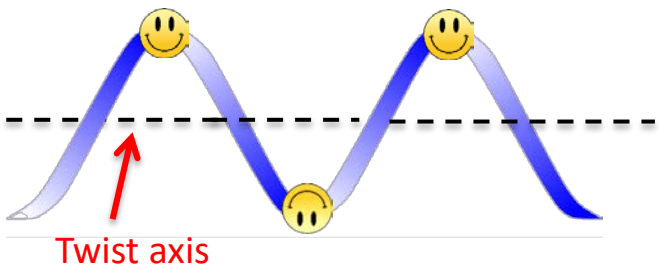


Measured results



M. Ebrahimpouri, E. Rajo-Iglesias, Z. Sipus and O. Quevedo-Teruel, "Cost-Effective Gap Waveguide Technology Based on Glide-Symmetric Holey EBG Structures," in *IEEE Trans. Microw. Theory Tech.*, vol. 66, no. 2, pp. 927-934, Feb. 2018

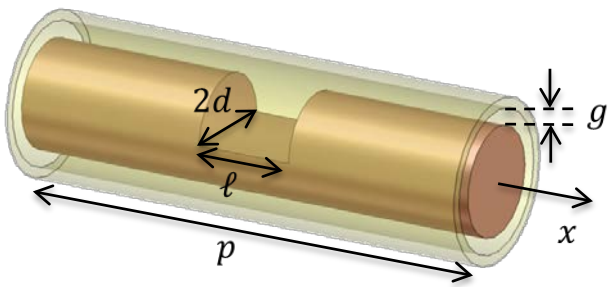
Twist-Symmetric Periodic Structures



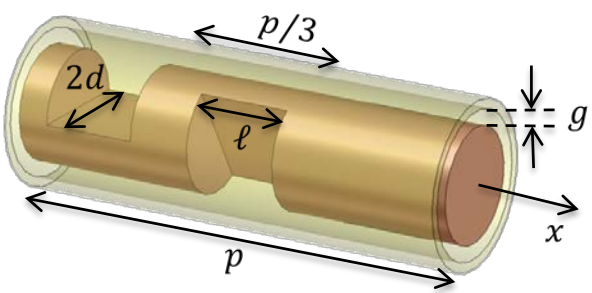
m-fold twist symmetry

$$S_m: \begin{cases} x \to x + \dfrac{p}{m} \\ \varphi \to \varphi + \dfrac{2\pi}{m} \\ \quad \to -z \end{cases}$$

Single hole



3-fold twist



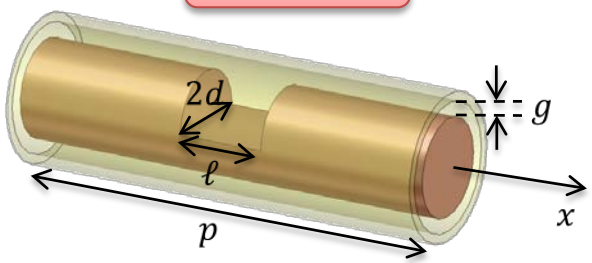
Eigenvalue problem for twist symmetries:

$$S_m[\boldsymbol{E}(\rho,\varphi,z)] = \boldsymbol{E}\left(\rho + \frac{p}{m}, \varphi + \frac{2\pi}{m}, z\right)$$
$$= e^{-jk_x\frac{p}{m}}\,\boldsymbol{E}(\rho,\varphi,z)$$

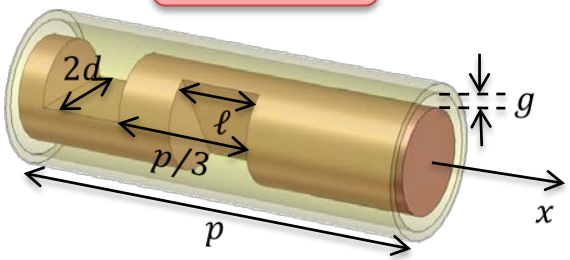
$k_x p$ is periodic of $2m\pi$

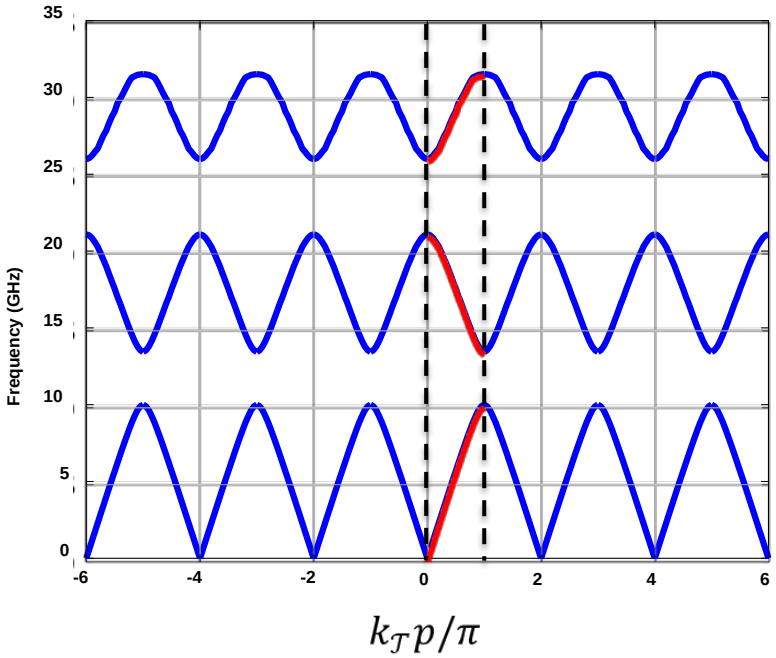
Twist Symmetry: Dispersion Diagrams

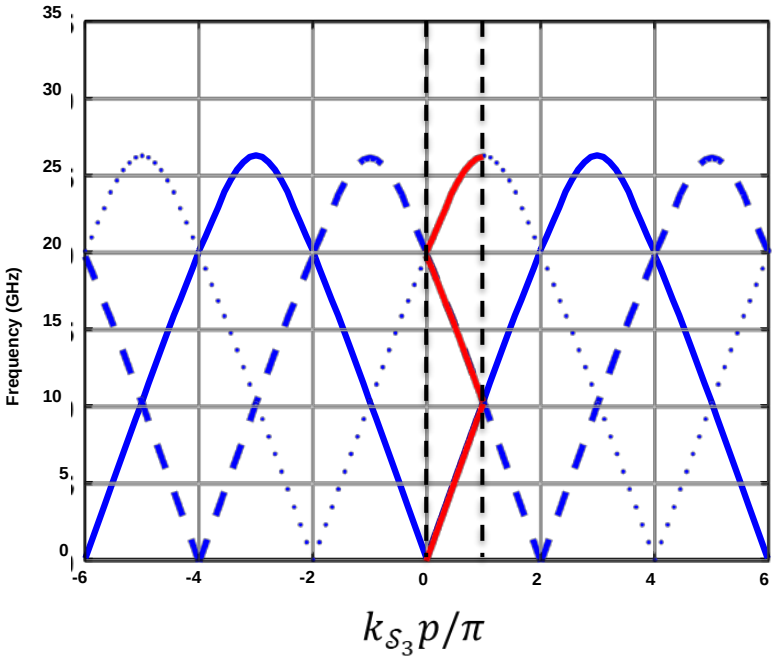
Single hole



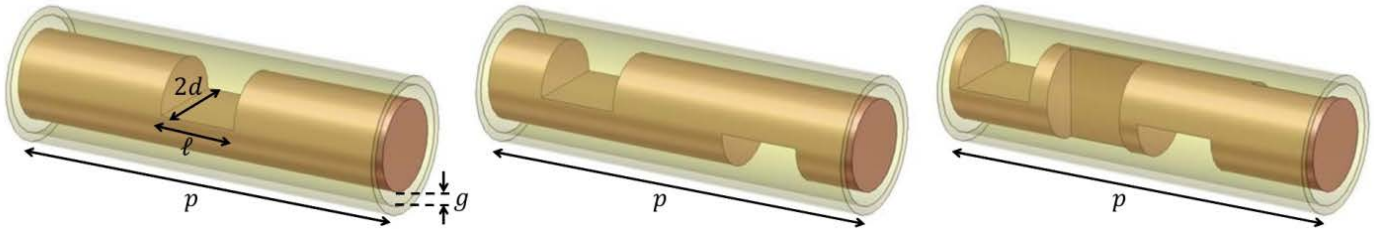
3-fold twist







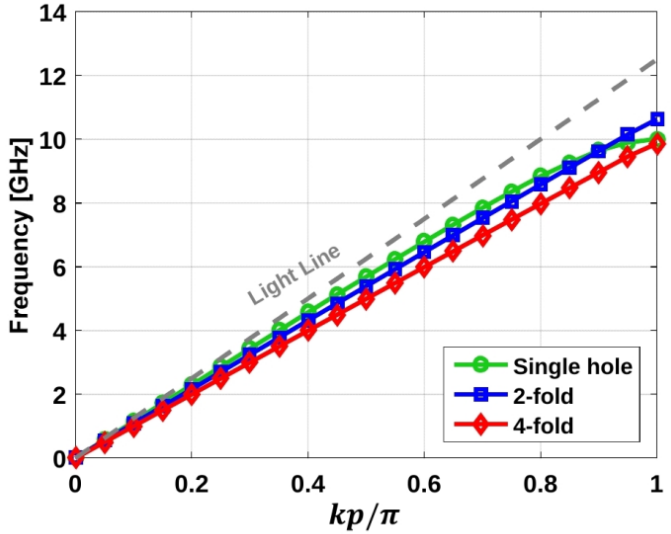
Twist Symmetry Effect



Single hole

2-fold twist

4-fold twist

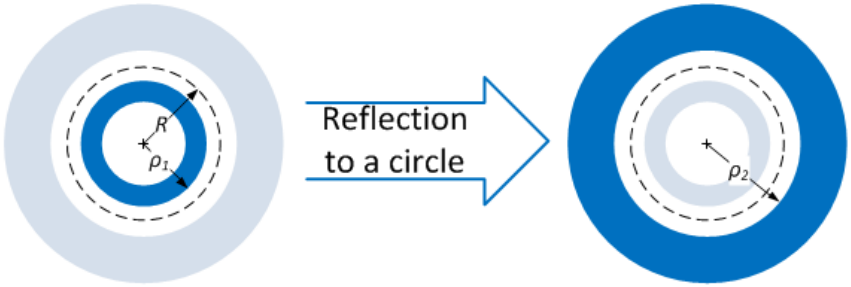


$$d = 1.5\ \text{mm},\ \ \ell = 2.4\ \text{mm},\ \ p = 12\ \text{mm},\ \ g = 0.1\ \text{mm}$$

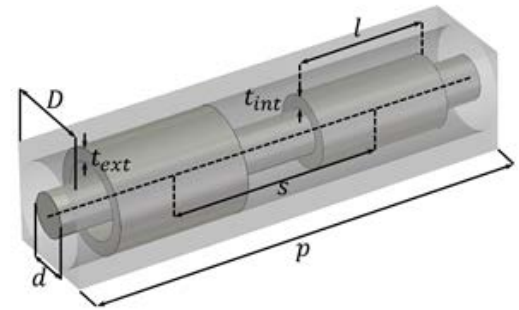
F. Ghasemifard, M. Norgren, and O. Quevedo-Teruel, "Twist and polar glide symmetries: an additional degree of freedom to control the propagation characteristics of periodic structures," *Scientific Reports*, vol. 8, Art. no. 11266, 2018.

Polar-Glide Symmetry

- Coaxial cable with rings inside and outside metallic conductors.
- Similar approach of transformation optics.

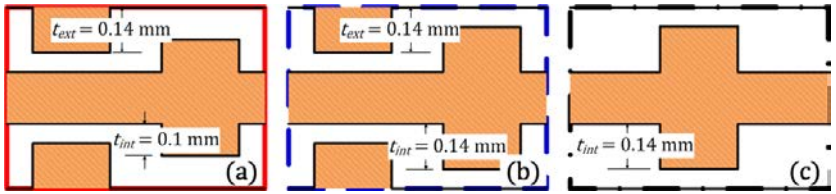


Applying polar glide symmetry to periodic structures



Polar glide-symmetric structure remains invariant under a translation and a reflection to a circular or cylindrical surface

$$S_{PG} \equiv \begin{cases} \rho \rightarrow \dfrac{R^2}{\rho} \\ \theta \rightarrow \theta + \alpha \\ z \rightarrow z + \dfrac{p}{2} \end{cases}$$



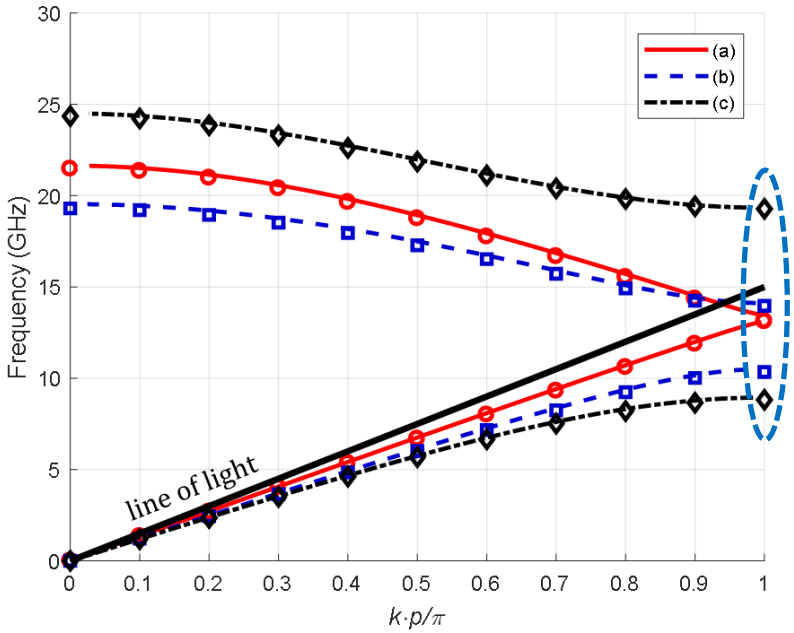
longitudinal section

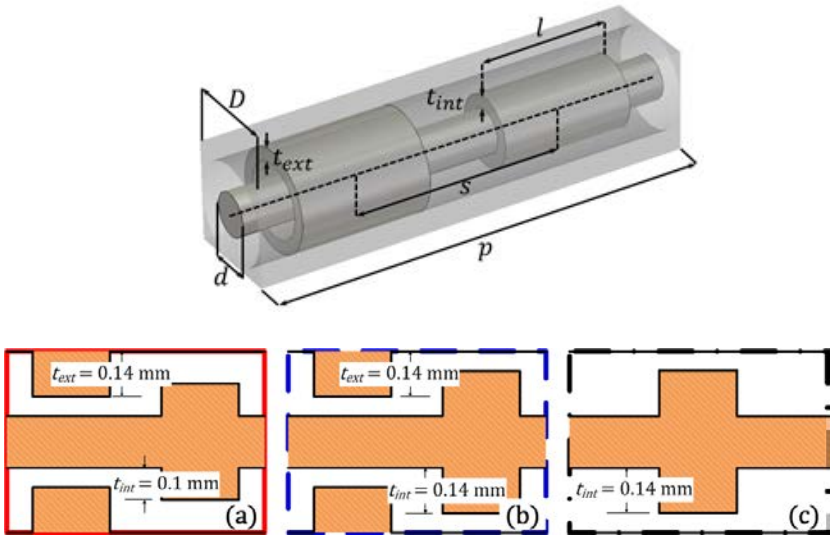
Q. Chen, F. Ghasemifard, G. Valerio, O. Quevedo-Teruel, "Modeling and Dispersion Analysis of Coaxial Lines with Higher Symmetries" *IEEE Transactions on Microwave Theory and Techniques*, vol. 66, no. 10, pp. 4338-4345, Oct. 2018.

Polar-Glide Symmetry

- Control of the stop-band in the propagation of the coaxial cable.

- Elimination of the first band-gap with polar gide.





Q. Chen, F. Ghasemifard, G. Valerio, O. Quevedo-Teruel, "Modeling and Dispersion Analysis of Coaxial Lines with Higher Symmetries" *IEEE Transactions on Microwave Theory and Techniques*, vol. 66, no. 10, pp. 4338-4345, Oct. 2018.

Summary

- ❑ Review of Maxwell equations under planar symmetries.
- ❑ Definition of glide (1-D and 2-D) and twist symmetries.
- ❑ Dispersion properties of higher order symmetries in terms of harmonic coupling
- ❑ Properties of higher-order symmetric structures: dispersion and stopbands
- ❑ Polar-glide symmetries